Interval-valued Pythagorean Fuzzy Hamacher Geometric Weighted Averaging Operator and its Applications to Decision Making

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Abstract: Based on the pythagorean fuzzy sets and its operations, stillmore, with the integration the algebraic aggregation operators and Einstein aggregation operators, an interval-valued pythagorean fuzzy Hamacher geometric weighted averaging operator are devised. Moreover, an illustrative example is given to demonstrate their practicality and effectiveness of the proposed methods.

1. Introduction

Multiple criteria group decision making is one of the fundamental branches of the decision theory. With the appearance of the fuzziness and uncertainty which exists in the presentation of data information in the decision making, Atanassov [1] proposed the intuitionistic fuzzy set (IFS) which is described by a membership degree and a nonmembership degree, IFS have two memberships which reduce the fuzziness to some extent. Furthermore, Atanassov and Gargov [2] developed the interval-valued intuitionistic fuzzy set (IVIFS) in which the membership degree and nonmembership degree are extended to interval numbers. However, under some situations, the sum of membership degree and nonmembership degree exceeds 1. This is out of the researching scope of IFS(IVIFS) theory. By losing this limitation that the sum of membership degree and nonmembership degree is less than 1,Yager[3] first proposed the concept of Pythagorean fuzzy set(PFS) which is characterized by the that its square sum of membership degree and nonmembership degree is not greater than 1.Later, Chen[4] developed a novel VIKOR-based methods for multiple criteria decision making based on Pythagorean fuzzy information. Yager [5] developed various aggregation operators to aggregate PFNs and IVPFNs

The remainder of this paper is structured as follows. Section 2 reviews some basic concepts of IVPFNs and Hamacher *t*−conorm and *t*−norm. In Section 3, we proposed Hamacher operations on IVPFNs and develop Hamacher geometric aggregation operators based on IVPFNs. In Section 4, an example is provided to demonstrate the procedure of application.

2. Basic Concepts

2.1 Interval-Valued Pythagorean Fuzzy Set.

Definition 1. Let X be a set, an interval valued pythagorean fuzzy set (IVPFS) P in X is defined as

$$P = \{\langle x, M_P(x), N_P(x)\rangle \mid x \in X\}$$

Where the intervals $M_P(x)$ and $N_A(x)$ represent, respectively, the membership degree and nonmembership degree of the element x to the set A. For each $x \in X$, and $M_P(x)$ and $N_P(x)$ are closed intervals and their lower and upper end points are, respectively, denoted by. $M_{PL}(x)$ and $M_{PU}(x)$, $N_{PL}(x)$ and $N_{PU}(x)$,and

$$0 \le N^2{}_{PL}(x) + N^2{}_{PU}(x) \le 1.$$

Thus, an IVPFS P in X is expressed by

$P = \{\langle x \mid [M_{PL}(x), M_{PU}(x)], [N_{PL}(x), N_{PU}(x)]\rangle \mid x \in X\}$, Where $0 \le N^2{}_{PL}(x) + N^2{}_{PU}(x) \le 1$.

Definition 2. α=<[a,b][c,d]> be a Pythagorean fuzzy number if $b^2 + d^2 \le 1$

To evaluate the performance of α, the score is defined as

Definition 3. Let α=<[a,b][c,d]> be a Pythagorean fuzzy number**.** The score of is defined as

$$S(\alpha)=(a^2-b^2+c^2-d^2)/2$$

2.2 Interval-Valued Pythagorean Fuzzy Set Induced by Hamacher Operators

In fuzzy set theory, some operations are defined to discover the relationships between the fuzzy sets. While union and intersection are two representive operations of fuzzy set, it is listed as

$$x\otimes y=xy, x\oplus y=x+y-xy$$

Furthermore, as a generalization of the above two operations, several *t*–norm and *t*–conorm are proposed. By adding a parameter *r*, Hamacher operations which include Hamacher sum and Hamacher product, they are

$$T(x,y)=\frac{xy}{r+(1-r)(x+y-xy)}, S(x,y)=\frac{x+y-xy-(1-r)xy}{1-(1-r)xy}$$

Definition 4.

Let

$$P=\{\langle x\,|\,[M_{PL}(x),M_{PU}(x)],[N_{PL}(x),N_{PU}(x)]\rangle\,|\,x\in X\}, Q=\{\langle x\,|\,[M_{QL}(x),M_{QU}(x)],[N_{QL}(x),N_{QU}(x)]\rangle\,|\,x\in X\}, n>0$$

be any two IVPFNS, then, the newly defined generalized intersection and union of P,Q are defined as follows:

$$(1) P\otimes Q=\langle[(S((M_{PL}(x))^2,S((M_{QL}(x))^2)^{1/2},(S((M_{PL}(x))^2,S((M_{QL}(x))^2)^{1/2}],$$
$$[T((N_{PL}(x))^2,T((N_{QL}(x))^2)^{1/2},(T((N_{PL}(x))^2,T((N_{QL}(x))^2)^{1/2}]\rangle,$$

$$(2) P\oplus Q=\langle[(T((M_{PL}(x))^2,T((M_{QL}(x))^2)^{1/2},(T((M_{PL}(x))^2,T((M_{QL}(x))^2)^{1/2}],$$
$$[S((N_{PL}(x))^2,S((N_{QL}(x))^2)^{1/2},(S((N_{PL}(x))^2,S((N_{QL}(x))^2)^{1/2}]\rangle,$$

$$(3) nP=\left\langle\left[\sqrt{\frac{(1+(r-1)(M_{PL}(x))^2)^n-(1-(M_{PL}(x)^2))^n}{(1+(r-1)(M_{PL}(x)^2)^n+(r-1)(1-(M_{PL}(x)^2)^n}},\sqrt{\frac{(1+(r-1)(M_{PU}(x)^2)^n-(1-(M_{PU}(x)^2)^n}{(1+(r-1)(M_{PU}(x)^2)^n+(r-1)(1-(M_{PU}(x)^2)^n}}\right],\right.$$
$$\left.\left[\frac{\sqrt{r}(N_{PL}(x))^n}{\sqrt{(1+(r-1)(1-(N_{PL}(x))^2)^n+(r-1)((N_{PL}(x)^2)^n}},\frac{\sqrt{r}(N_{PU}(x))^n}{\sqrt{(1+(r-1)(1-(N_{PU}(x)^2))^n+(r-1)((N_{PU}(x)^2)^n}}\right]\right\rangle,$$

$$(4)\alpha^n=\left\langle\left[\frac{\sqrt{r}(M_{PL}(x))^n}{\sqrt{(1+(r-1)(1-(M_{PL}(x))^2)^n+(r-1)((M_{PL}(x)^2)^n}},\frac{\sqrt{r}(M_{PU}(x))^n}{\sqrt{(1+(r-1)(1-(M_{PU}(x)^2))^n+(r-1)((M_{PU}(x)^2)^n}}\right]\right\rangle,$$
$$\left[\sqrt{\frac{(1+(r-1)(N_{PL}(x))^2)^n-(1-(N_{PL}(x)^2))^n}{(1+(r-1)(N_{PL}(x)^2)^n+(r-1)(1-(N_{PL}(x)^2)^n}},\sqrt{\frac{(1+(r-1)(N_{PU}(x)^2)^n-(1-(N_{PU}(x)^2)^n}{(1+(r-1)(N_{PU}(x)^2)^n+(r-1)(1-(N_{PU}(x)^2)^n}}\right]\rangle$$

When *r* = 1, Hamacher *t*-norm and *t*-conorm are reduced to algebraic *t*-norm, when *r* = 2, Hamacher *t*-norm and *t*-conorm are reduced to Einstein *t*-norm.

3. Interval-valued Pythagorean Hamacher Geometric Weighted Averaging Operator.

Definition 5. Let **Θ** be the set of IVPFNs, $P_i=\{\langle x\,|\,[M_{P_iL}(x),M_{P_iU}(x)],[N_{P_iL}(x),N_{P_iU}(x)]\rangle\,|\,x\in X\}$ be the set of IVPFNs, where $w=(w_1,w_2,\cdots,w_n)^T$ is the weight vector of $P_i(i=1,2,\cdots,n)$ then, with $w_1\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$, and let IVPFGHWA: $\Theta^n\to\Theta$, if $IVPFGHWA_w(P_1,P_2,\cdots,P_n)=(P_1)^{w_1}\otimes(P_2)^{w_2}\otimes\cdots\otimes(P_n)^{w_n}$ Then, IVPFGHWA is called the interval-valued Pythagorean fuzzy Hamacher geometric weighted averaging operator.

Theorem. Let **Θ** be the set of IVPFNs, $P_i=\{\langle x\,|\,[M_{P_iL}(x),M_{P_iU}(x)],[N_{P_iL}(x),N_{P_iU}(x)]\rangle\,|\,x\in X\}$ be the set of IVPFNs, where $w=(w_1,w_2,\cdots,w_n)^T$ is the weight vector of $P_i(i=1,2,\cdots,n)$ then, with $w_1\in[0,1]$, $\sum_{i=1}^{n}w_i=1,$ then, the aggregating result from Definition 4 is an IVPFN.

$$IVPFGHWA_w(P_1,P_2,\cdots,P_n)=$$

$$\left\langle\left[\frac{\sqrt{r}\prod_{i=1}^{n}(M_{P_iL}(x))^{w_i}}{\sqrt{\prod_{i=1}^{n}(1+(r-1)(1-(M_{P_iL}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(M_{P_iL}(x)^2)^{w_i}}},\frac{\sqrt{r}\prod_{i=1}^{n}(M_{P_iU}(x))^{w_i}}{\sqrt{\prod_{i=1}^{n}(1+(r-1)(1-(M_{P_iU}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(M_{P_iU}(x)^2)^{w_i}}}\right]\right\rangle,$$

$$\left[\sqrt{\frac{\prod_{i=1}^{n}(1+(r-1)(N_{P_iL}(x))^2)^{w_i}-\prod_{i=1}^{n}(1-(N_{P_iL}(x)^2))^{w_i}}{\prod_{i=1}^{n}(1+(r-1)(N_{P_iL}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(1-(N_{P_iL}(x)^2))^{w_i}}},\sqrt{\frac{\prod_{i=1}^{n}(1+(r-1)(N_{P_iU}(x))^2)^{w_i}-\prod_{i=1}^{n}(1-(N_{P_iU}(x)^2))^{w_i}}{\prod_{i=1}^{n}(1+(r-1)(N_{P_iU}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(1-(N_{P_iU}(x)^2))^{w_i}}}\right]\Bigg\rangle$$

4. Example

Table 1. Matrix given by expert E_1

Attribute	D_1	D_2	D_3
1	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.7], [0.4,0.5])	P ([0.7,0.9], [0.3,0.4])
1	P ([0.6,0.7], [0.3,0.5])	P ([0.6,0.8], [0.4,0.5])	P ([0.7,0.9], [0.2,0.3])
1	P ([0.5,0.6], [0.4,0.5])	P ([0.7,0.8], [0.4,0.5])	P ([0.6,0.7], [0.3,0.4])
1	P ([0.4,0.6], [0.3,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.8], [0.4,0.5])
Attribute	D_4	D_5	D_6
1	P ([0.7,0.8], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.4,0.6], [0.5,0.7])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.6], [0.1,0.3])	P ([0.8,0.9], [0.1,0.3])
1	P ([0.7,0.9], [0.1,0.3])	P ([0.4,0.6], [0.2,0.3])	P ([0.7,0.8], [0.4,0.5])
1	P ([0.6,0.8], [0.4,0.5])	P ([0.5,0.6], [0.1,0.3])	P ([0.8,0.9], [0.2,0.3])

Table 2. Matrix given by expert E_2

Attribute	D_1	D_2	D_3
1	P ([0.6,0.7], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])	P ([0.5,0.7], [0.2,0.5])
1	P ([0.5,0.7], [0.4,0.5])	P ([0.5,0.6], [0.3,0.5])	P ([0.5,0.8], [0.4,0.6])
1	P ([0.5,0.6], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.6], [0.4,0.5])
1	P ([0.7,0.9], [0.1,0.3])	P ([0.7,0.8], [0.4,0.5])	P ([0.6,0.8], [0.4,0.5])
Attribute	D_4	D_5	D_6
1	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.7], [0.2,0.4])	P ([0.5,0.7], [0.4,0.5])
1	P ([0.5,0.6], [0.1,0.3])	P ([0.6,0.8], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])
1	P ([0.6,0.7], [0.4,0.6])	P ([0.8,0.9], [0.2,0.3])	P ([0.6,0.7], [0.4,0.6])
1	P ([0.4,0.6], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])	P ([0.5,0.7], [0.4,0.5])

With the proposed method, we choose the optimal high-tech enterprise with the lowest risk of innovation from four candidate enterprises $\{A_1, A_2, A_3, A_4\}$.The criteria the evaluation of technologic innovation are follows:D_1: Policy risk; D_2:Financial risk; D_3: Technological risk; D_4: Production risk;D_5:Market risk;D_6:Managerial risk. The three decision makers(experts) $\{E_1,E_2,E_3\}$

who are specializing in risk evaluation fields are invited to evaluate these four high-tech enterprises according to the six evaluation criteria $\{D_1,D_2,D_3,D_4,D_5,D_6\}$.The weight vector is put in advance as $\omega = (0.4,0.35,0.25)$, and the weight vector of criteria is:

$$w = (0.1894,0.1841,0.1361,0.1257,0.1753,0.1894).$$

The entries values of alternatives with respect to attribute are presented in Table 1,2,3.

Table 3. Matrix given by expert E_3

Attribute	D_1	D_2	D_3
1	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])	P ([0.5,0.7], [0.2,0.5])
1	P ([0.6,0.7], [0.1,0.3])	P ([0.4,0.6], [0.1,0.3])	P ([0.8,0.9], [0.2,0.3])
1	P ([0.6,0.8], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.8], [0.4,0.5])	P ([0.6,0.8], [0.4,0.5])
Attribute	D_4	D_5	D_6
1	P ([0.6,0.8], [0.3,0.4])	P ([0.6,0.8], [0.4,0.5])	P ([0.6,0.7], [0.4,0.5])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.8], [0.2,0.5])	P ([0.8,0.9], [0.2,0.3])
1	P ([0.6,0.8], [0.4,0.5])	P ([0.6,0.7], [0.4,0.6])	P ([0.7,0.8], [0.4,0.5])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.4,0.5], [0.4,0.6])	P ([0.5,0.6], [0.4,0.5])

Aggregate the vectors, when r=5, compute the scores with Definition 2, we have $S(r_1)=0.3540, S(r_2)=0.3908, S(r_3)=0.3270, S(r_4)=0.3622,$ The order is, $S(r_2)>S(r_4)>S(r_1)>S(r_3)$. Finally, we obtain $A_2 \succ A_4 \succ A_1 \succ A_3$. Then, the best alternative is A_3. This result coincides with [6].

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